The Parabola

The basic parabola is $\mathbf{y=x^2}$ other graphs of this type are just movements of this basic shape.
With knowledge of their movements you should be able to sketch the graph without having to draw up a table first. However, if worst comes to worst drawing up a table and plotting points is a good method to draw any graph.

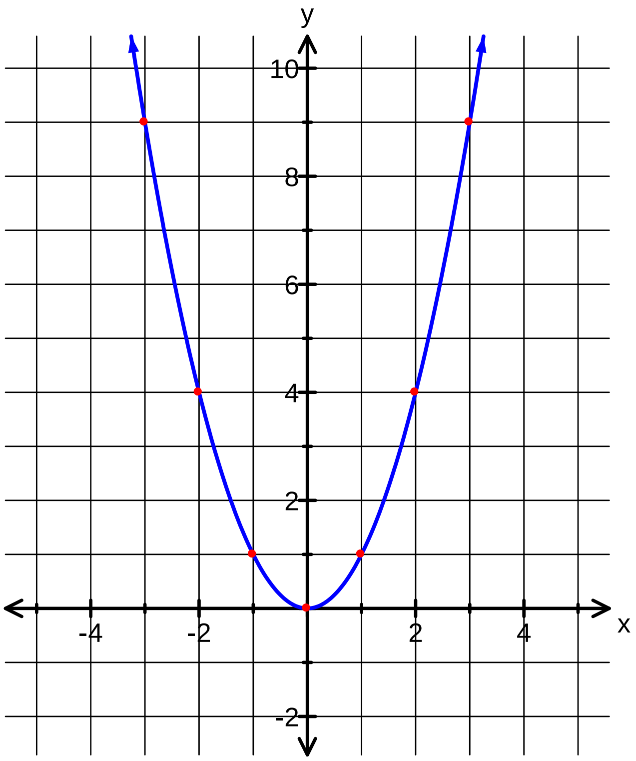
Example: *the basic parabola* $\mathbf{y=x^2}$

x	-3	-2	-1	0	1	2	3
$y=x^2$	$y=(-3)^2$ $y=9$	$y=(-2)^2$ $y=4$	$y=(-1)^2$ $y=1$	$y=(0)^2$ $y=0$	$y=(1)^2$ $y=1$	$y=(2)^2$ $y=4$	$y=(3)^2$ $y=9$
Coordinate to plot	(-3,9)	(-2,4)	(-1,1)	(0,0)	(1,1)	(2,4)	(3,9)

Notice how the values of y are all positive, this is as any number squared results in a positive. This means when we graph the points the graph will be above the x axis.

To Graph

- Plot the points (remember "x is across" and "y is up/down")
- Join with a smooth curve – this is not a straight line graph, needs to have a curved bottom



<u>Key Features:</u>

Vertex – The turning point of the graph, in this example it is (0,0)

A parabola is a *symmetrical* graph about the line that goes through the vertex. This graph is symmetrical about the y-axis.

Follows the pattern of:
Out 1 from the vertex, up 1
Out 2, up 4 since $2^2=4$
Out 3, up 9 since $3^2=9$
Out 4, up 16… and so on following the pattern

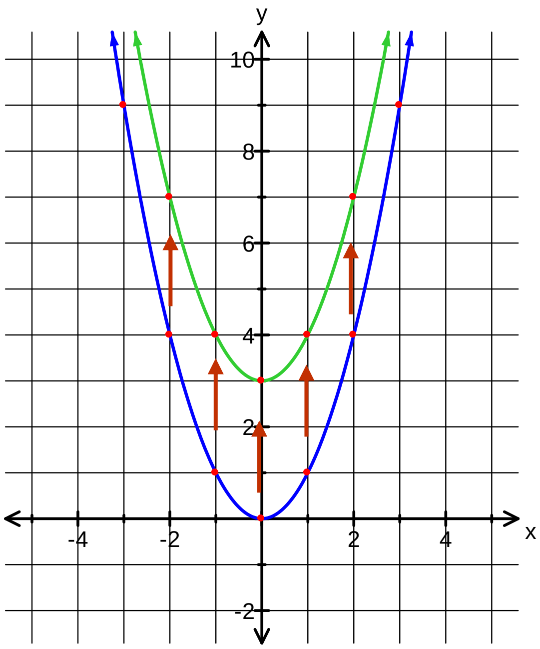
Out **a** from the vertex, up $\mathbf{a^2}$

Movements of Parabola - Vertical (up/down)

When the equation is in the form: $y = x^2 + a$

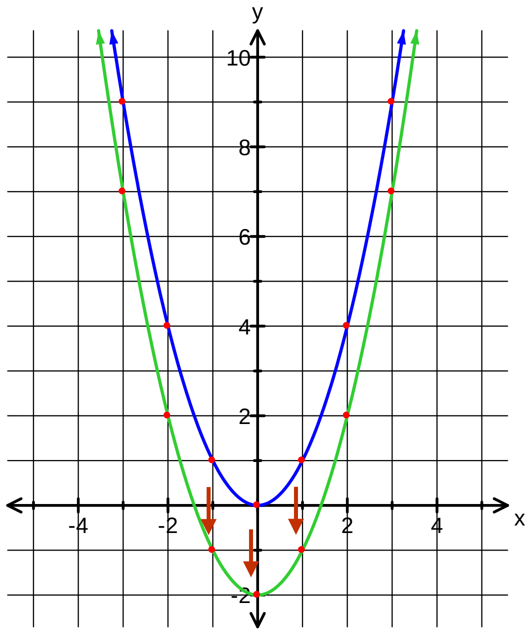
Adding a number will shift the graph *up* by **a** units

e.g. $y=x^2 + 3$ *parabola moves up by 3*



Subtracting shift the graph *down* by **a** units

e.g. $y=x^2 - 2$ *parabola moves down by 2*



Try to graph these and state new vertex:

1. $y=x^2 + 4$

   New vertex is at (,)
   The graph moves up/down by?

2. $y=x^2 - 1$

   New vertex is at (,)
   The graph moves up/down by?

3. $y=x^2 + 1$

   New vertex is at (,)
   The graph moves up/down by?

HINT: if your stuck try drawing up a table of values to finds points to plot

i.e.

x	$y=x^2 + a$
-2	
-1	
0	
1	
2	

Remember **vertex = "turning point"**

Movements of Parabola – Horizontal (left/right)

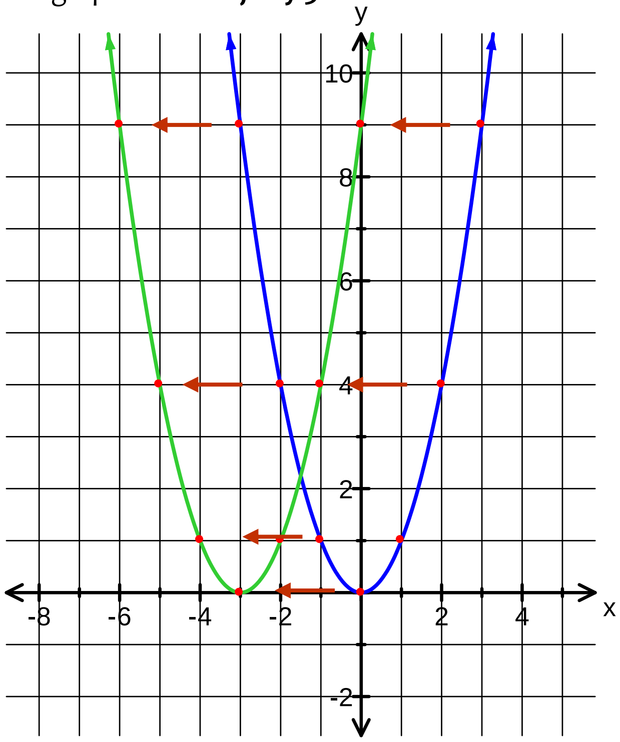
When the equation is in the form: $\boxed{y = (x + a)^2}$

NOTE: *we find the x intercept when y=0, so we want to find a value of x that will make the bracket = 0 (This will just be the opposite of the value of a, that is why the graph moves the opposite way than what you may think.)*

Adding inside the bracket moves the graph left (negative direction)

e.g. $y=(x+3)^2$

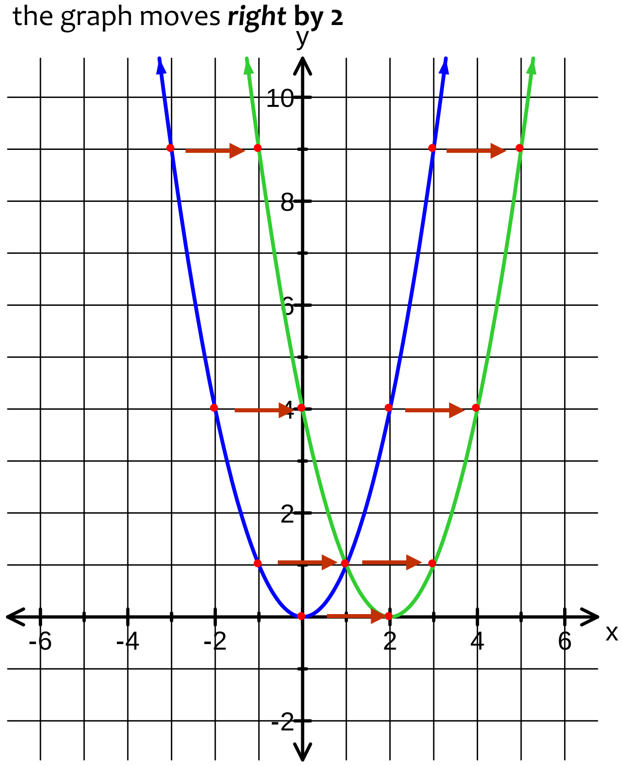
The graph moves ***left* by 3**



Subtracting inside the brackets moves the graph right (positive direction)

e.g. $y=(x-2)^2$

the graph moves ***right* by 2**



Try to graph these and state new vertex:

1. $y=(x+2)^2$
   New vertex is at (,)
   The graph moves left/right by?

2. $y=(x-4)^2$
   New vertex is at (,)
   The graph moves left/right by?

3. $y=(x+4)^2$
   New vertex is at (,)
   The graph moves left/right by?

HINT: if your stuck try drawing up a table of values to finds points to plot

Choose values for your table of x close to the opposite of the number in the equations

Remember parabolas are symmetrical about the turning point

Vertex will be the opposite of the number

Parabola in factorised form – the intercept method

When the equation is in the form: **$y=(x+a)(x+b)$** **DO NOT EXPAND!!!**

When y=0 we can find our x intercepts
equations in this form will have 2 x-intercepts one when (x+a)=0 and another when (x+b)=0

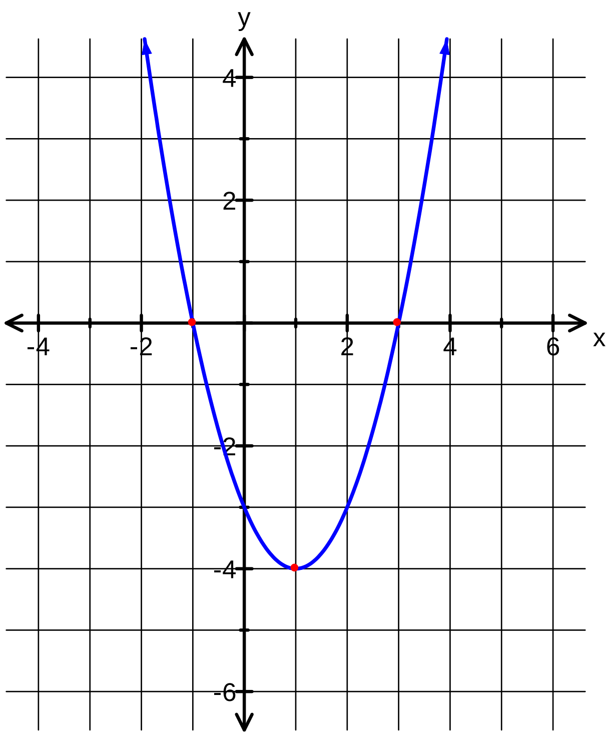
EXAMPLE: $\boldsymbol{y = (x-3)(x+1)}$

1. To find **x-intercepts** we set y=0 0=(x-3)(x+1) *this is true if* **x-3=0** or **x+1=0**

   So x-intercepts are $\boldsymbol{x = 3}$ and $\boldsymbol{x = -1}$

2. **Vertex:** is halfway between so find by adding x-intercepts together and dividing by 2
   (3+(-1))÷2=1
   So our vertexs x-coordinate will be at x=1
   To find the y coordinate we substitute x=1 into the equation
   y=(1-3)(1+1)
   y=(-2)×2
   y=-4
   The coordinates of our vertex are (1,-4)

Knowing our ***x-intercepts*** and the ***vertex*** we can sketch the graph:



Remember the pattern:

Out **1** from the vertex up **1**
since $1^2=1$
Out 2 from the vertex up **4**
since $2^2=4$
Out 3 from the vertex up **9**
since $3^2=9$

Changing the steepness

When the equation is in the form: $\boxed{y=ax^2}$

If there is a number in front of the x^2 it will either make the graph steeper or flatter
When the number is negative it flips the graph so it is upside down

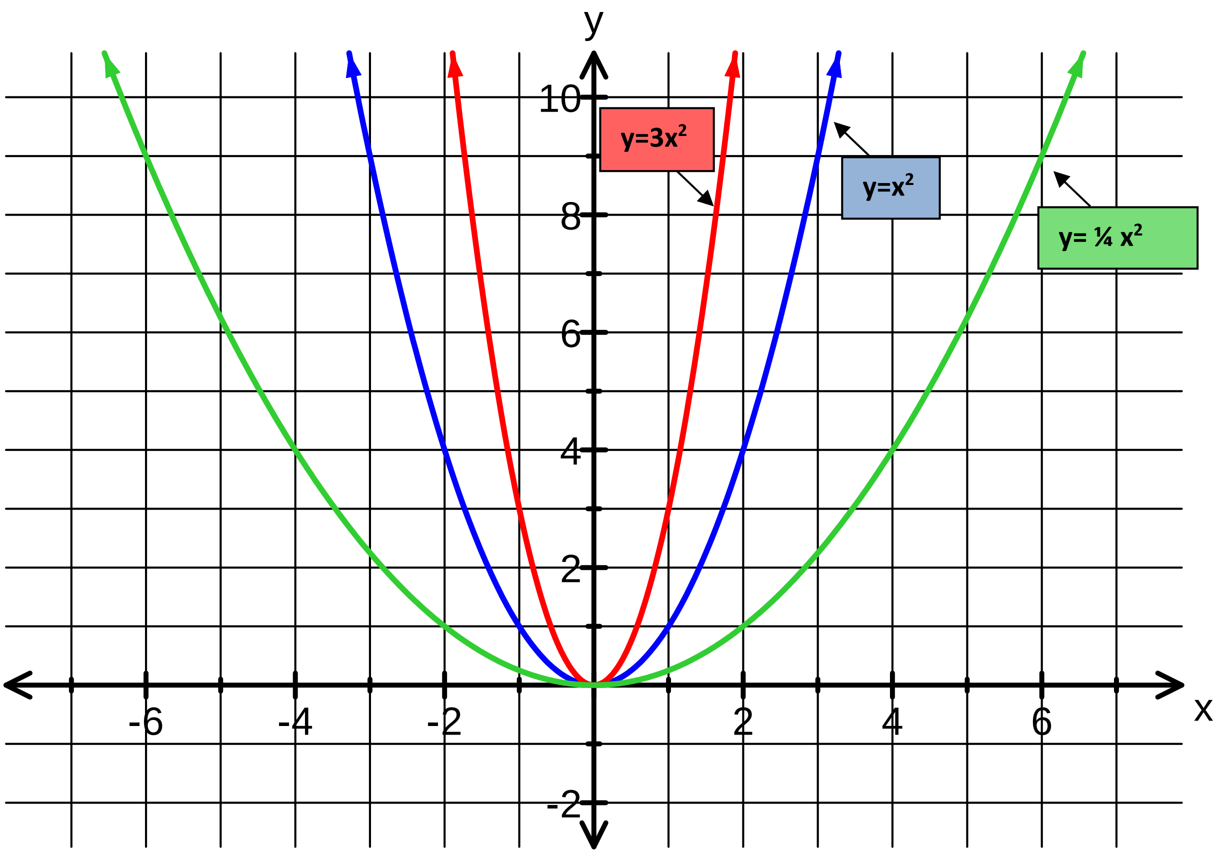
- If the number in front is BIGGER than 1
  e.g. $3x^2$ means "3 times the x value squared"

  it makes the parabola **steeper** than the basic $y=x^2$

- if the number in front is smaller than 1
  e.g. $¼x^2$ means "one quarter of the x value squared"

  it makes the parabola **flatter** than the basic $y=x^2$

EXAMPLES:



Parabola summary

For parabolas you need to know how to do the following

- **Graph parabolas of the form:**
  - $y = (x + a)^2$ — *sideways movement*
  - $y = x^2 + b$ — *up/down movement*
  - $y = (x+a)^2 + b$ — **Vertex method**
    moves the vertex up/down AND sideways
  - $y = (x+a)(x+b)$ — ***x-intercept method***
  - $y = ax^2$ — The coefficient changes *the steepness of the graph*

- **Identify the key features**
  - x-intercepts — *Found when y=0*
  - vertex — *"turning point" middle of graph*
  - y-intercepts — *Found when x=0*